Performance Optimization of the Magneto-hydrodynamic Generator at the Scramjet Inlet

Nilesh V. Kulkarni

Advisors:

Prof. Minh Q. Phan
Dartmouth College

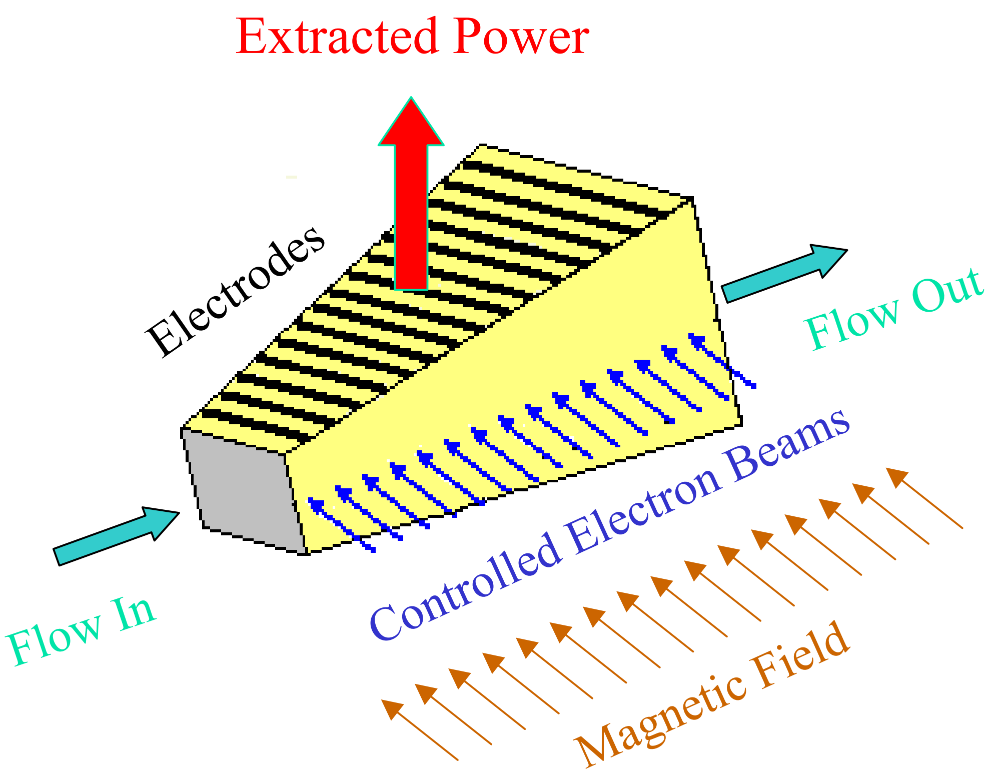
Prof. Robert F. Stengel
Princeton University

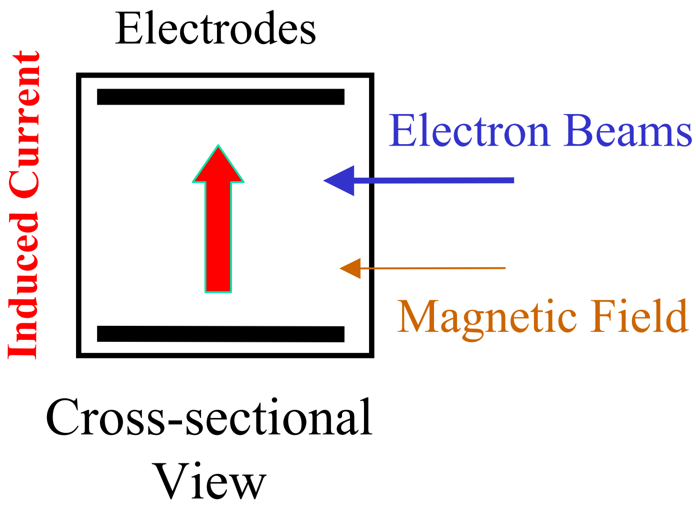
Joint University Program Meeting
Cambridge, MA
17th October - 18th October 2002

Presentation Outline

- The Magneto-hydrodynamic (MHD) generator
- The role of control
- MHD generator system
- Cost-to-go design for optimal control using neural networks
- Results
- Conclusions

Magneto-Hydrodynamic (MHD) Generator at the Inlet





Schematic of the MHD Generator

MHD Generator System

- Assumptions
  - One-dimensional steady state flow
  - Inviscid flow
  - No reactive chemistry
  - Low Magnetic Reynolds number
- *x*-*t* equivalence

Flow Equations

- Continuity Equation

$$\frac{d(\rho u A)}{dx} = 0$$

x - Coordinate along the channel

ρ - Fluid density

u - Fluid velocity

A - Channel cross-section area

- Force Equation

$$\rho u \frac{du}{dx} + \frac{dP}{dx} = -(1-k)\sigma u B^2$$

P - Fluid pressure

k - Load factor

σ - Fluid conductivity

B - Magnetic field

Flow Equations…

- Energy Equation

$$\rho u \frac{d(\gamma\varepsilon + \frac{u^2}{2})}{dx} = -k(1-k)\sigma u^2 B^2 + Q_\beta$$

ε - Fluid internal energy

Q_β - Energy deposited by the e-beam

- Continuity Equation for the electron number density

$$\frac{d(n_e u)}{dx} = \frac{2 j_b \varepsilon_b}{eY_i Z} - \beta n_e^2$$

n_e - Electron number density

j_b - Electron beam current

ε_b - E-beam energy

Z - Channel width

Y - Ionization potential

Performance Characterization

$$J = p_1\left[T(x_f) - T_e\right]^2 + p_2\left[M(x_f) - M_e\right]^2 + \\ + \int_0^{x_f}\left[\begin{array}{l}\dfrac{q_1}{\rho u A}\left[Q_\beta A - k(1-k)\sigma u^2 B^2 A\right] + \\ q_2 h(P) + q_3 dS^2 + r_1 j_b^2\end{array}\right]dx$$

- Attaining prescribed values of flow variables at the channel exit (Mach number, Temperature)
- Maximizing the net energy extracted which is the difference between the energy extracted and the energy spent on the e-beam ionization
- Minimizing adverse pressure gradients
- Minimizing the entropy rise in the channel
- Minimizing the use of excessive electron beam current

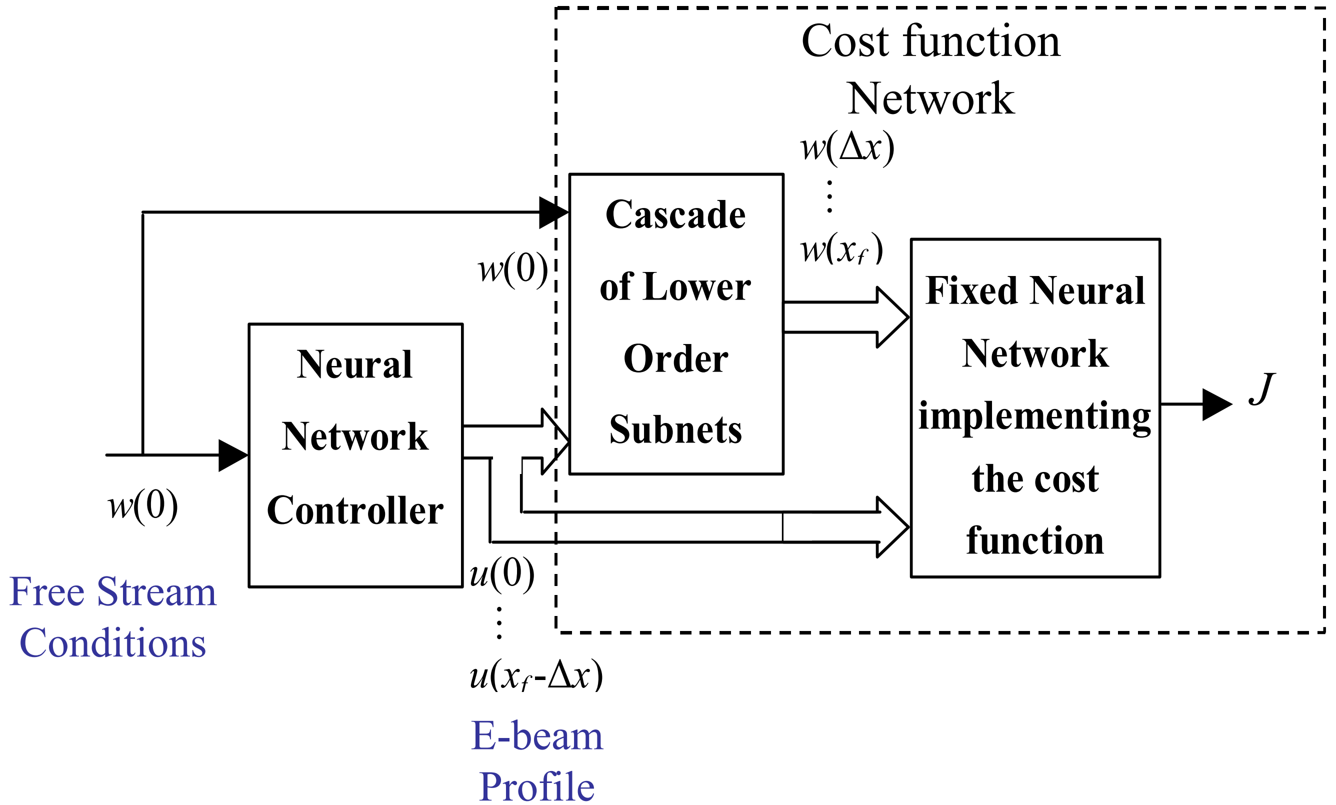
The Predictive Control Based Approach for Optimal Control

- Features of our optimal controller design technique
  - Works for both linear and nonlinear systems
  - Data-based
  - Finite horizon, end-point optimal control problem
  - Equivalent to time (position) varying system dynamics

[1] Kulkarni, N.V. and Phan, M.Q., "Data-Based Cost-To-Go Design for Optimal Control," *AIAA Paper* 2002-4668, *AIAA Guidance, Navigation and Control Conference*, August 2002.

[2] Kulkarni, N.V. and Phan, M.Q., "A Neural Networks Based Design of Optimal Controllers for Nonlinear Systems," *AIAA Paper* 2002-4664, *AIAA Guidance, Navigation and Control Conference*, August 2002.

Optimal Control Using Neural Networks



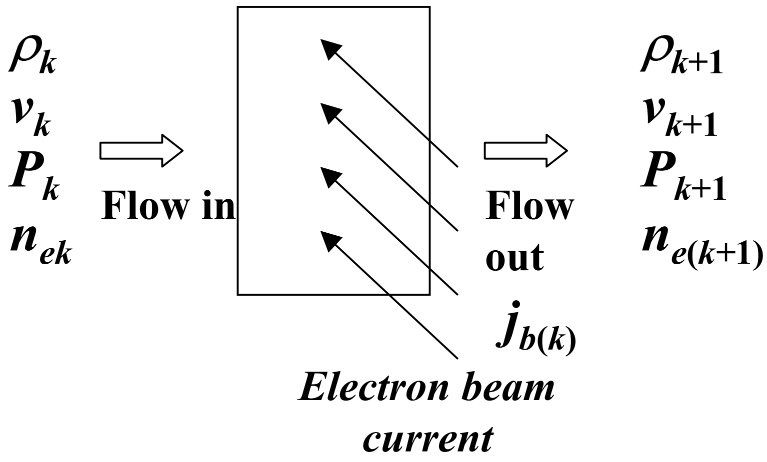
Optimal control architecture

Formulation of the Control Architecture: Cost Function Approximator

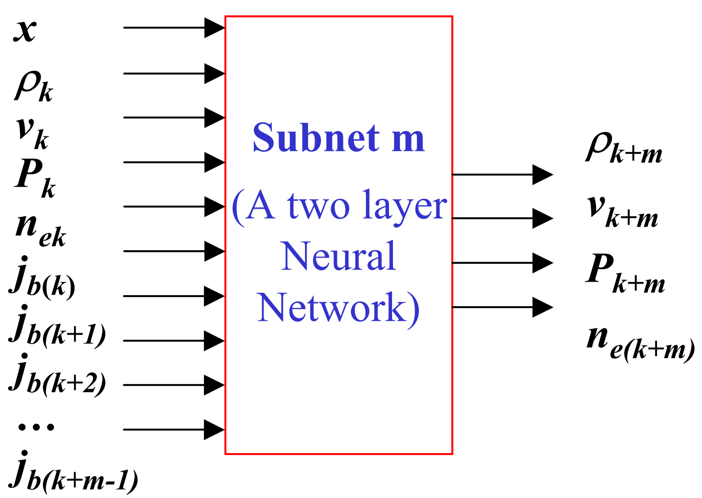
- Collecting system data through simulation or a physical model
- Parameterizing single step ahead and multi-step ahead models called subnets using neural networks
- Training the subnets using system data
- Formulating a fixed layer neural network that take the subnet outputs and calculate the cost-to-go function or the cumulative cost function.

Using Subnets to Build the Cost Function Network

- Continuously spaced e-beam windows each having a length of 0.5 cm
- Subnet 1 chosen to correspond to the system dynamics between a group of 4 e-beam windows
- Length of the channel = 1 m
- Need subnets up to order 50

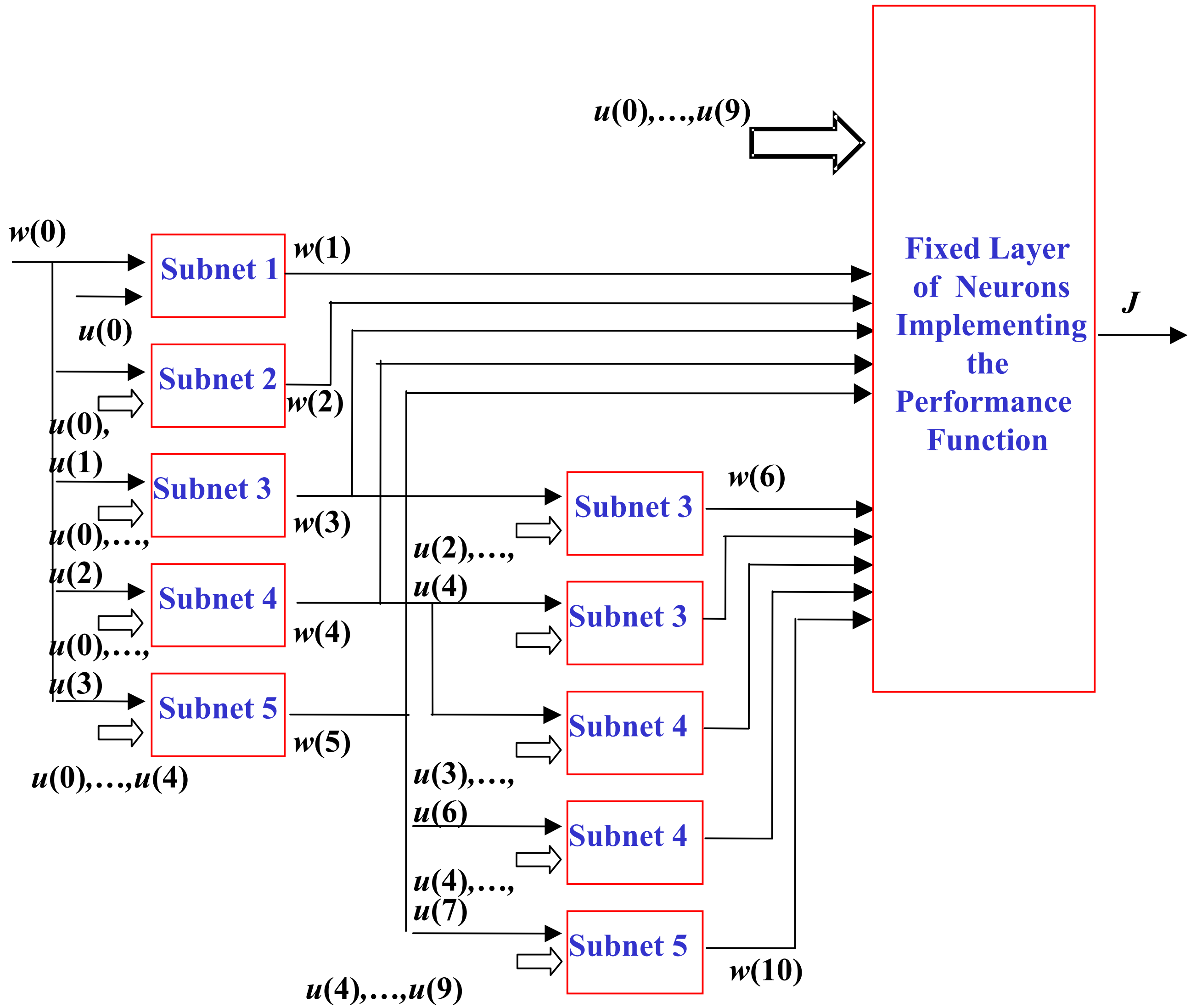


Physical picture describing Subnet 1



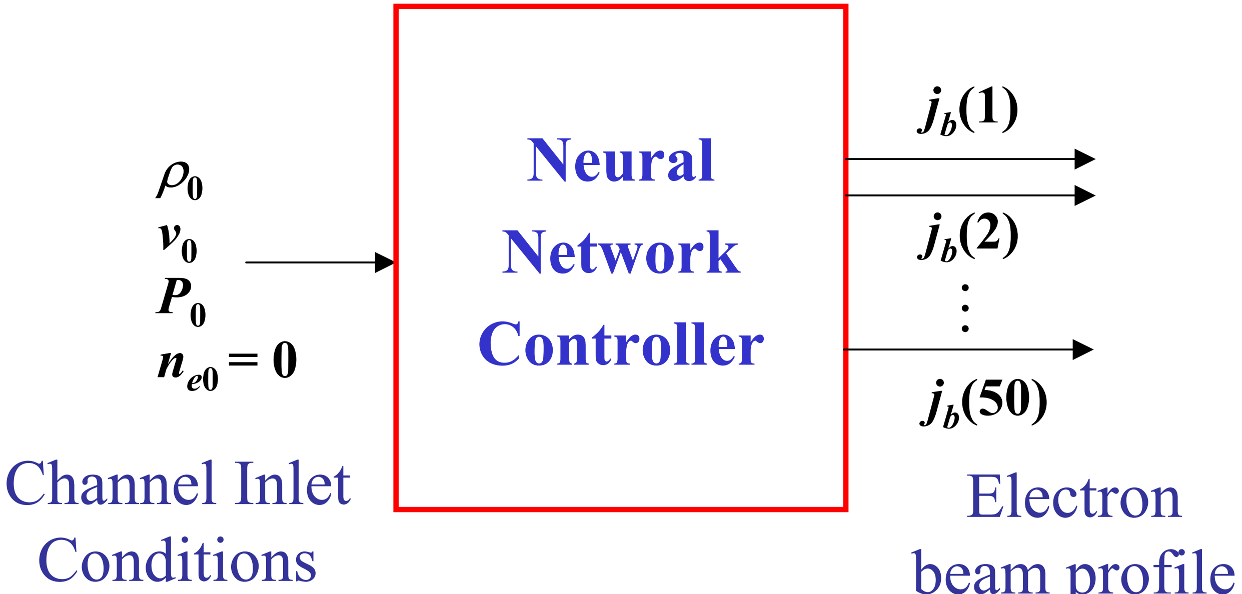
Subnet *m*, inputs and outputs.

Cost Function Network

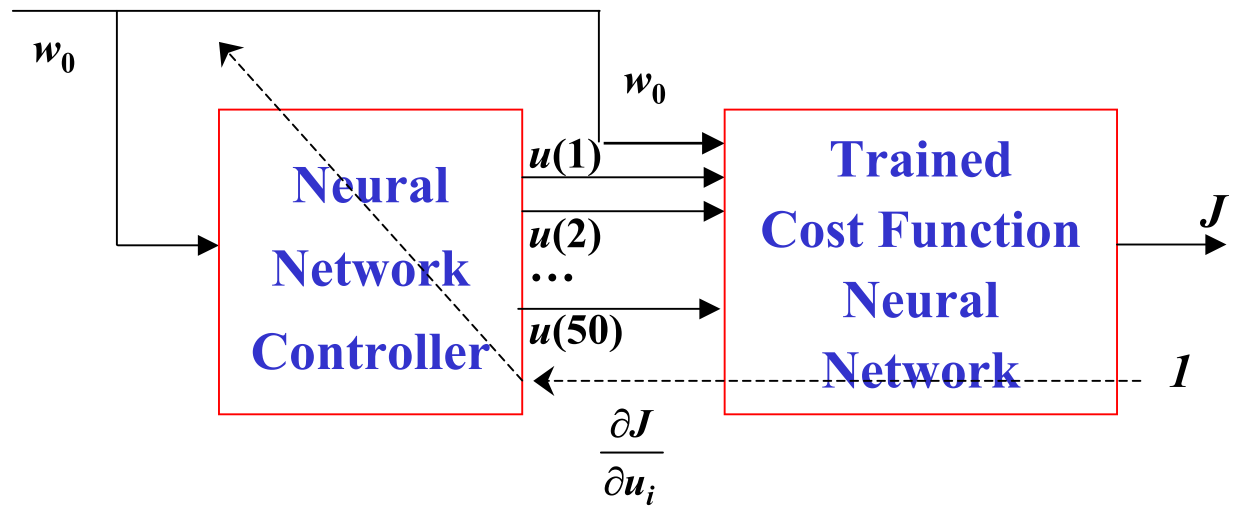


Implementation of the Cost function network of order r = 10, using trained subnets of order 1 through 5

Formulation of the Control Architecture: Neural Network Controller



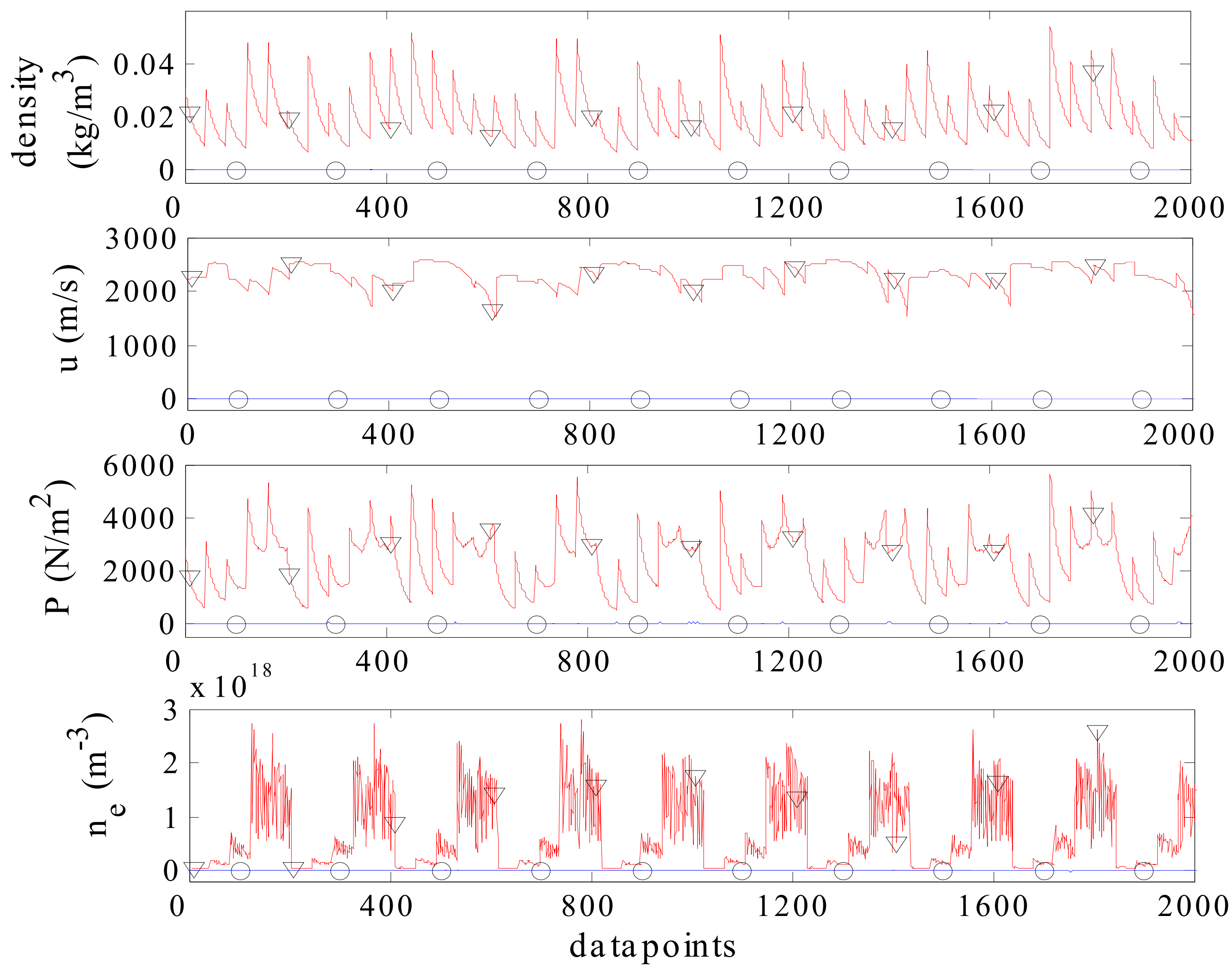
Neural Network Controller Training



- Gradient of J with respect to the control inputs $u(1)$,…, $u(50)$ is calculated using back-propagation through the *CGA* neural network.
- These gradients can be further back-propagated through the neural network controller to get, $\frac{\partial J}{\partial W_{nn}}$ (W_{nn} - weights of the network)
- Neural network controller is trained so that

$$\frac{\partial J}{\partial W_{nn}} \to 0$$

Training Results for Subnet 10



Testing Subnet 10, '∇ '- Ouput value given by subnet 10, 'o' – Error between the subnet 10 output and the actual value given by the simulation

Case 1: Maximizing the Net Power Extracted

Cost function:

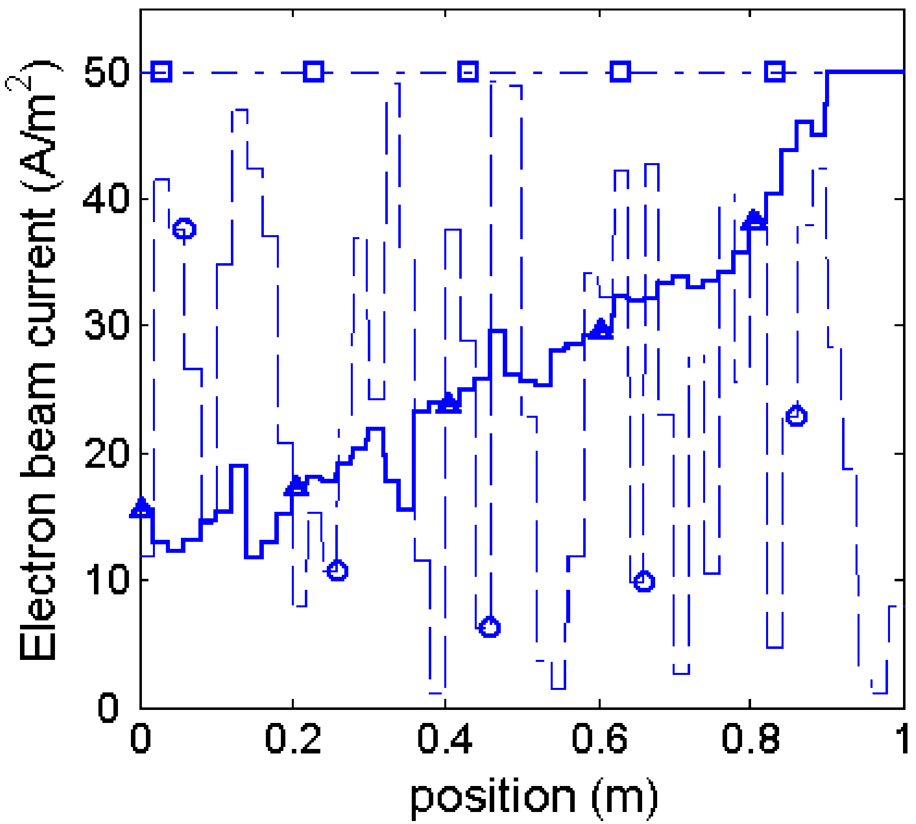
$$J = p_1\left[T(x_f) - T_e\right]^2 + p_2\left[M(x_f) - M_e\right]^2 +$$

$$+\sum_{i=1}^{50}\left[\begin{array}{l}\dfrac{q_1}{\rho(i)u(i)A(i)}\left[Q_\beta(i)A(i) - k(1-k)\sigma(i)u(i)^2B^2A(i)\right] + \\ q_2(i)P(i) + q_3[S(i) - S(i-1)]^2 + r_1 j_b(i-1)^2\end{array}\right]\Delta x$$

p_1	p_2	q_1	q_2	q_3	r_1
0	0	0.0001	0	0	0.005

Power input-output for the three control profiles

h=30 km, M=8	Power Spent	Power Extracted	Net Power Extracted
Constant current (50 A/m^2)	300 kW	1.918 MW	1.618 MW
Random current	121 kW	1.381 MW	1.260 MW
Optimal Profile	174 kW	1.717 MW	1.544 MW



Electron beam current profile □ - constant e-beam current (50 A/m^2), O- random profile, Δ - neural network controller.

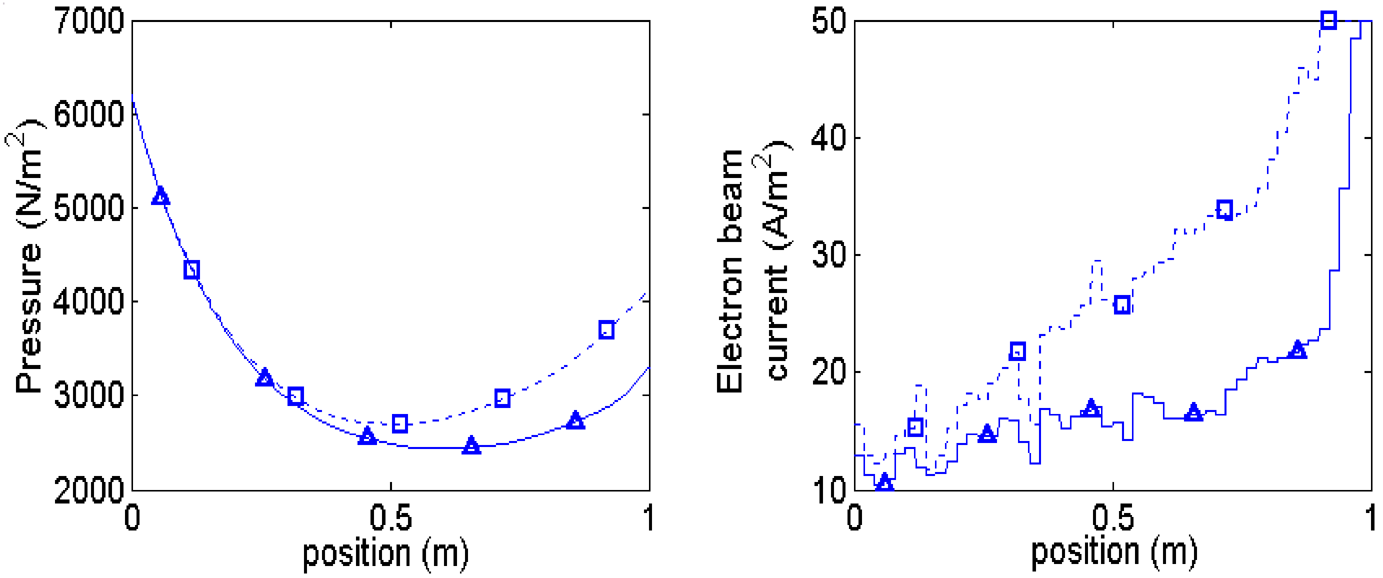
Case 2: Imposing Pressure Profile Penalty

Choice of the weighting parameters in the cost function:

p_1	p_2	q_1	q_3	r_1
0	0	0.0001	0	0

$$q_2(x) = 0; \quad 0 < x < 0.9$$

$$q_2(x) = 200\,x^4; \quad 0.9 < x < 1$$

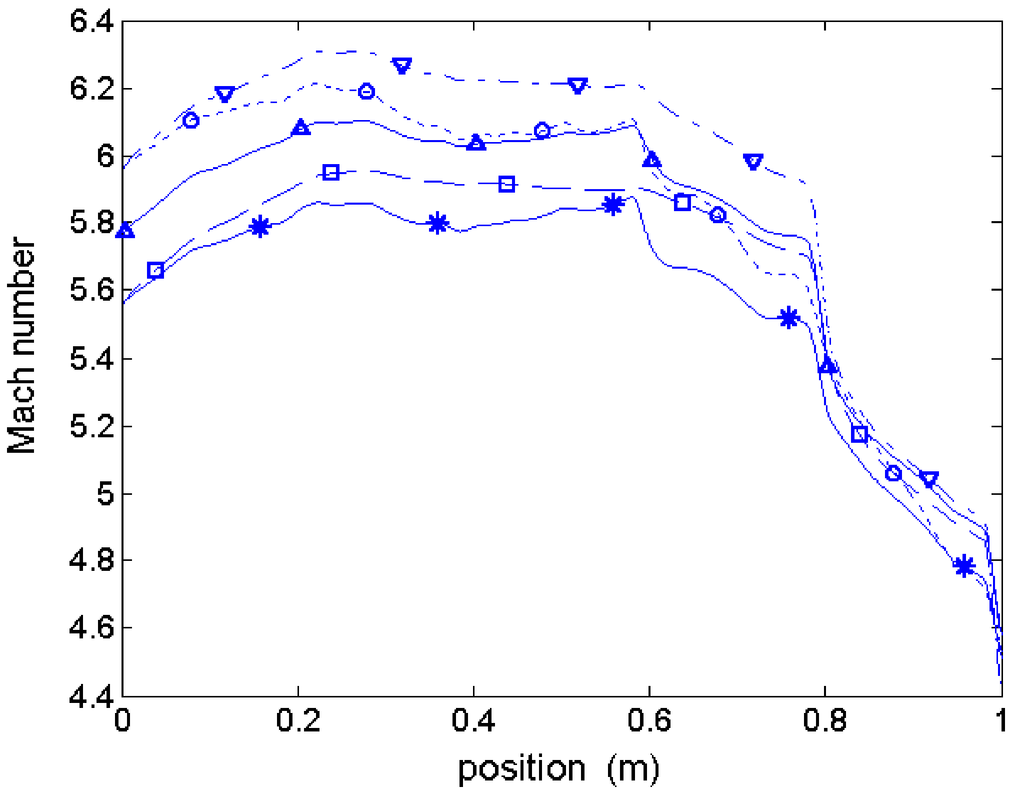


E-beam current profile and the resulting pressure distribution along the channel, □- without pressure weighting, Δ - with pressure weighting.

Case 3: Prescribing an Exit Mach Number

Choice of the weighting parameters in the cost function:

p_1	p_2	q_1	q_2	q_3	r_1
0	100	10^{-6}	0	0	0



Mach number profiles for different free stream conditions

Prescribed Exit Mach Number

$M_e = 4.5$

Free Stream Altitude	Free Stream Mach number	Exit Mach number	Legend in the plots
30 km	8	4.41	—— Δ
31.5 km	7.6	4.58	- - - - □
28.5 km	8.4	4.52	. . . . . O
31.5 km	8.4	4.51	-.-.-.-∇
28.5 km	7.6	4.51	—— *

Conclusions

- Formulation of the problem of performance optimization of the MHD Generator as an optimal control problem
- Implementation of the cost-to-go design approach for optimal control using neural networks
- Data-based approach
- Successful implementation for different performance criteria
- Future work to incorporate sensors along the channel to further optimize the system performance